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FRACTIONAL TUNE AND LINEAR RESONANCES IN UMER

INTRODUCTION:

I. Resonances, BPM and beam spectrum:

Generally, the betatron resonance condition in a circular machine such as a synchrotron or storage ring is expressed by

$$n\nu_X + m\nu_y = Np, \tag{1}$$

where *n*, *m*. *p* and *N* are integers, and ν_x and ν_y are the horizontal and vertical bare tunes (the notation Q_X, Q_Y is used in Europe). *N* is the super-periodicity of the machine, and |n| + |m| is the order of the resonance. In UMER, *N* = 1.

The most destructive resonances are the *linear resonances*, i.e. the first and second order resonances. The first order resonances are associated with dipole errors, while the second order resonances are caused by quadrupole (i.e. gradient) errors. The names *integer and half-integer resonances* are also employed to describe first and second order resonances.

As explained in the first reference below, a beam position monitor (BPM) is sensitive to the *linear dipole-moment density,* or

$$d \propto \cos(q\omega_0 t) + \sum_{n'=1}^{\infty} \cos[(n' \pm q)\omega_0 t], \tag{2}$$

where ω_0 is the (angular) revolution frequency of the beam in the ring, *q* is the *fractional part of the bare tune* (horizontal or vertical) and *n' = k + M* is an index containing an integer *k* and the integer part, *M*, of the bare tune. The beam spectrum in Eq. (2) contains harmonics of the revolution frequency through $n'\omega_0$; it also contains frequency components called *"fast" and "slow" waves* corresponding to the *+q* and *–q* terms on the right hand side, and a low-frequency line at $q\omega_0$. It can be seen that a single BPM can only detect the fractional part of the tune.

II. Tune vs. Quadrupole Current:

It is important for the experiment to understand the relation between bare tune and quadrupole gradient (and applied current). Consider a simple FODO cell as in Fig. 3.27 in Prof. M. Reiser's book. Straightforward matrix multiplication (see, for example, P. Schmüser, in CERN Accelerator School Proceedings 87-10, edited by S. Turner, Geneva, 1987) leads to the following equation for the focusing strength of the thin-lens (and equivalent hardtop) quadrupoles:

$$\kappa_{qe} = 4\frac{Sin(\sigma_0/2)}{Sl}, \quad (3)$$

where S = 32 cm is the full-lattice period , σ_0 is the zero-current (or "undepressed") phase advance per period, and l = 5.164 cm is the effective length of the regular UMER quadrupole. If ν_o is the bare tune, and N = 36 is the number of FODO cells, then $\sigma_0 = 2\pi\nu_o/36$ (in radians). The relation connecting horizontal bare tune for 10 keV electrons in UMER and the quadrupole current follows from g_e = 2.60 Gauss/cm - amp (effective quadrupole gradient per amp):

$$\nu_0 = 3.64\times I_q(Amp), \quad (4)$$

where I_q is the quadrupole current. This relation is correct to within a few percent.

BACKGROUND:

Mario Serio, "Transverse Betatron Tune Measurements", in Observation, Diagnosis and Correction Proceedings, Anacapri, Isola di Capri, Italy, 1988 (Lectures Notes in Physics 343, M. Month, S. Turner – Eds.).

For an introduction to resonances, we recommend Prof. M. Reiser's, book, Sec. 3.8.6, and D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators*, Sec. 3.4

EQUIPMENT:

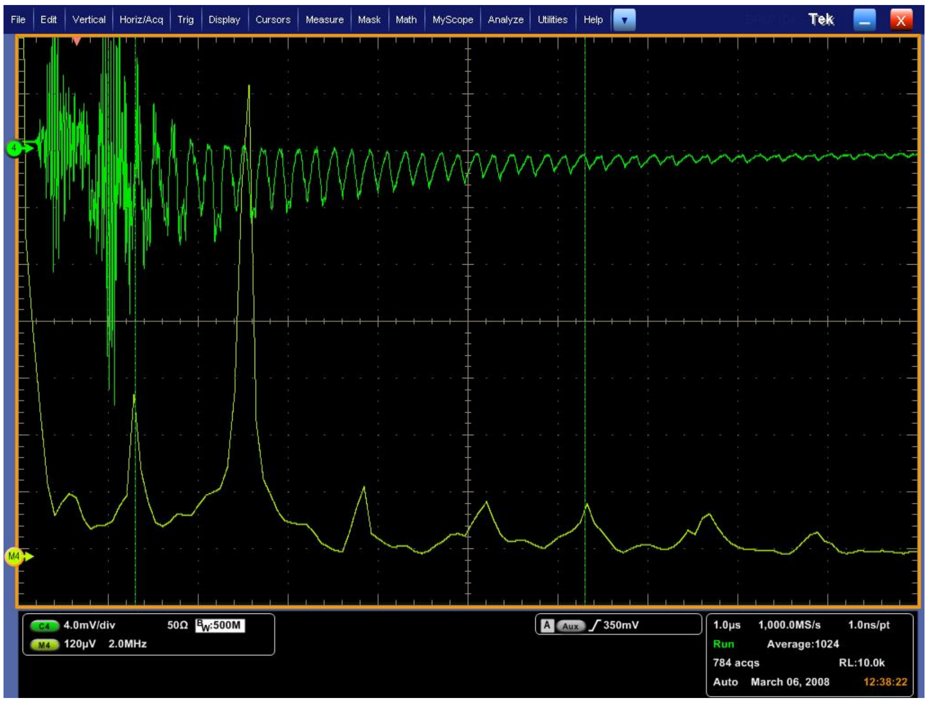
UMER, BPM, oscilloscope for BPM signal and FFT analysis, and the wall-current monitor at RC10.

PROCEDURE:

1. Start UMER control GUI at default settings for 7 mA at the "83%" operating point (ring quadrupole current = 1.826 A). Make sure you get at least 5 turns without noticeable beam losses, as measured by the wall-current monitor at RC10.
2. Select a BPM (RC1, 3 or 5 are best) and observe on the oscilloscope the signal from one of the horizontal BPM channels. On the scope, set your time scale to 1 μs/div, sampling rate to 1,000 MS/s. Make sure you use a transformer box to cancel the DC offset from the BPM amplifier and that the coupling into the scope is 50 Ω.
3. FFT Setup (in Tektronix DPO 7104 scope or similar): In the Math setup do "magnitude spectrum" linear. Choose center frequency = 10MHz,

frequency span = 20MHz, resolution bandwidth = 178kHz, window type = rectangular, position = -4,14 div, scale = 120μV.

4. Gating: It is important to restrict the signal to be Fourier-analyzed to a region outside the noisy part at the start (from pulser noise), and to cover at least 25 turns. Make sure you have the following gate setup (or something close): position = 3.15μs, duration 5.0 μs, length = 5000.
5. With the settings described above and quadrupole currents = 1.819 A, see if you can reproduce the signal shown below (one channel for the horizontal BPM plate and one math function for the FFT).



Notice the sidebands around the main frequency peak and the two vertical lines that indicate the gate for the FFT of the BPM signal on top. The horizontal freq. scale is 2 MHz/div.

6. Change the quadrupole currents in the ring from 1.826 A to 1.850 A, and monitor the beam current. Continue increasing the quad currents in steps of 0.025 A. At some point you will see a sudden reduction in beam current. Also monitor the evolution of the sidebands around the main peak. Record the quadrupole current (or current range) for minimum beam current
7. Continue increasing the quadrupole current in 0.025 A increments. At some point the beam current will be recovered. Record the quadrupole current at the maximum beam current point.
8. Go back to I_q near 1.800 A and reduce the quad current in steps of 0.025A while observing the 10^{th}-turn beam current peak. Do you detect a “dip” in the 10^{th}-turn peak of the total current signal? It may take a few trials to see the minimum. Try to record the quadrupole current corresponding to the drop in circulated current.

ANALYSIS / QUESTIONS:

1. Tabulate your results.
2. Calculate the revolution frequency in UMER for a 10 keV electron beam. The ring circumference is 11.52 m. Also calculate the first two harmonics and the sideband frequencies (around the main harmonic) corresponding to a fractional tune of 0.5.
3. Derive equation (4) above. What are the approximations involved?
4. What are the horizontal bare tunes corresponding to the two minima in the circulated beam current (use Eq. 4). Can you identify the type of resonances detected?
5. Linear resonances are independent of particle amplitude, while non-linear resonances are not. Discuss the reason for this difference.
6. The “stopband” is a measure of the resonance width. Qualitatively, comment on physical factors that lead to non-zero stopbands.
7. (Bonus) We may have cheated in using Eq. 4 to calculate bare tune, because the approximations may yield an error as large as 0.5! A more accurate FODO model would be needed. For a quadrupole current of say 1.826 A, calculate the bare tune using a thick-lens model for the FODO (see Eq. 3.354 in Prof. M. Reiser’s book).
8. (Bonus) Computer exercise in WinAgile. Instructions to be handed out separately.